Theory of Probability

A critical introductory treatment

Volume 1

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4.6 LIKELIHOOD

4.6.1. *Bayes's theorem*—in the case of events E, but not random quantities X—permits us to write $\mathbf{P}(\cdot|H)$ in the form we met above, a form which is often more expressive and practical:

$$\mathbf{P}(E|H) = \mathbf{P}(E)\mathbf{P}(H|E)/\mathbf{P}(H) = K \,.\, \mathbf{P}(E)\mathbf{P}(H|E), \tag{5}$$

where the normalizing factor, $1/\mathbf{P}(H)$, can be simply denoted by K, and, more often than not, can be obtained more or less automatically without calculating $\mathbf{P}(H)$. For this reason, it is often convenient to talk simply in terms of *proportionality* (i.e. by considering $\mathbf{P}(\cdot|H)$ only up to an arbitrary, non-zero, multiplicative constant, which can be determined, if necessary, by normalizing).

One could say that $\mathbf{P}(\cdot|H)$ is proportional to $\mathbf{P}(\cdot)$ and to $\mathbf{P}(H|\cdot)$, where the dot stands for E, thought of as varying over the set of all the events of interest. More concisely, this is usually expressed by saying that

'final probability' $=K$ 'initial probability' $\times$ 'likelihood', where $=K$ denotes proportionality, and we agree to call:

the *initial* and *final* probabilities those not conditional or conditional on H, respectively (i.e. evaluated before and after having acquired the additional knowledge in question, H), and

the *likelihood* of H given E, the $\mathbf{P}(H|E)$ thought of as a function of E (and possibly multiplied by any factor independent of E, e.g. $1/\mathbf{P}(H)$, the use of which would allow the substitution of '$=$' for '$=K$', or anything resulting from the omission of common factors, more or less cumbersome, or constant, or dependent on H). The term 'likelihood' is to be understood in the sense that a larger or smaller value of $\mathbf{P}(H|E)$ corresponds to the fact that the knowledge of the occurrence of E would make H either more or less probable (our meaning would be better conveyed if we spoke of the 'likelihoodization' of H by E).

4.6.2. This discussion leads to an understanding of how it should be possible to pass from the initial probabilities to the final ones through intermediate stages, under the assumption that we obtain, successively, additional pieces of information $H_1, H_2, \ldots, H_n$ (giving, altogether, $H = H_1H_2\ldots H_n$). In fact, one can also verify analytically that

$$\begin{aligned}
\mathbf{P}(E|H_1H_2) &= \mathbf{P}(EH_1H_2)/\mathbf{P}(H_1H_2)\\
&= [\mathbf{P}(E)\mathbf{P}(H_1|E)\mathbf{P}(H_2|EH_1)]/[\mathbf{P}(H_1)\mathbf{P}(H_2|H_1)]\\
&= K \,.\, \mathbf{P}(E) \,.\, \mathbf{P}(H_1|E) \,.\, \mathbf{P}(H_2|EH_1)
\end{aligned}$$

$=$ (the probability of E) $\times$ (the likelihood of H_1 given E) $\times$ (the likelihood of H_2 given EH_1).